Deep Inelastic Scattering and Parton Model

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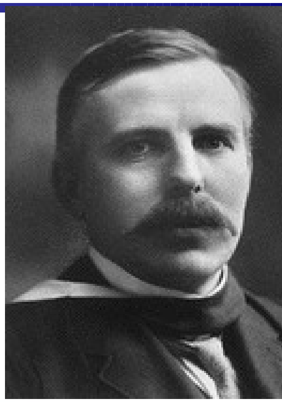
- References
  - G. Sterman, *Partons, Factorization and Resummation*, hep-ph/9606312
  - John Collins, *The Foundations of Perturbative QCD*, published by Cambridge, 2011
  - CTEQ, *Handbook of perturbative QCD*, Rev. Mod. Phys. 67, 157 (1995).
- General references
  - CTEQ web site: http://www.phys.psu.edu/~cteq/

Outline

- General Introduction: Brief History and Basics of Basics
- Deep Inelastic Scattering and Parton Model

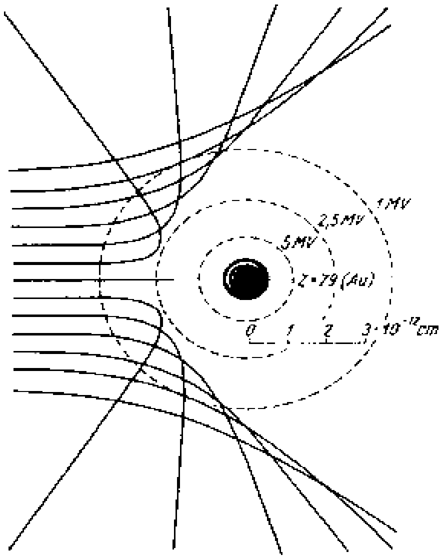
Rutherford scattering

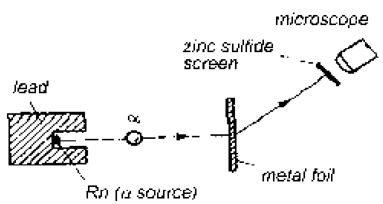


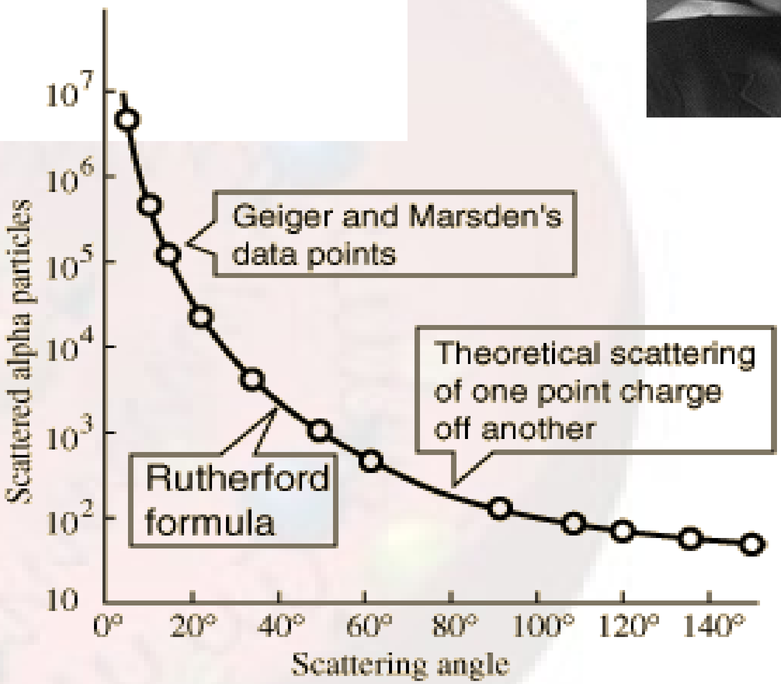
The Scattering of α and β Particles by Matter and the Structure of the Atom

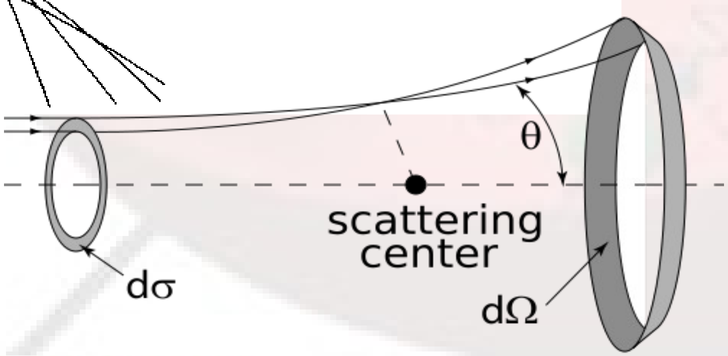
E. Rutherford, F.R.S.*

Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688



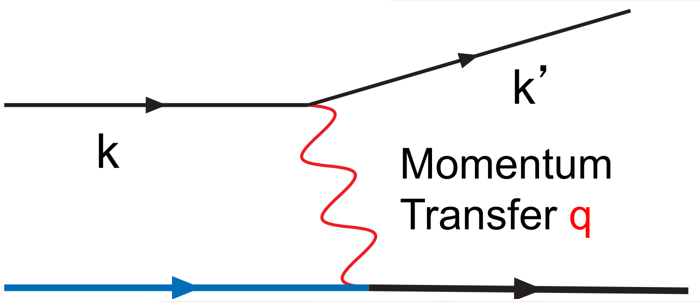






$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar c}{2mv_0^2}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Power counting analysis



$$2E_{k'}\frac{d\sigma}{d^3k'} \propto |\mathcal{M}|^2 \quad \mathcal{M} \propto \frac{1}{q^2}$$

$$q^2 = -Q^2 \approx E_k E'_k \sin^2\frac{\theta}{2}$$

- EM interaction perturbation, leading order dominance, potential~1/r
- Point-like structure
- Powerful tool to study inner structure

Basic idea of nuclear science

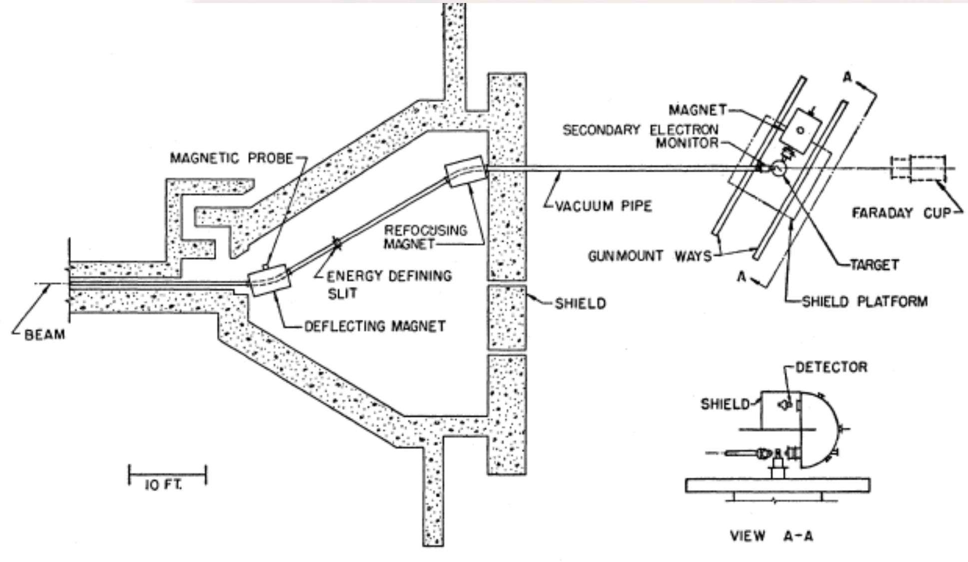
Since the α and β particles traverse the atom, it should be possible from a close study of the nature of the deflexion to form some idea of the constitution of the atom to produce the effects observed. In fact, the scattering of high-speed charged particles by the atoms of matter is one of the most promising methods of attack of this problem. The develop-

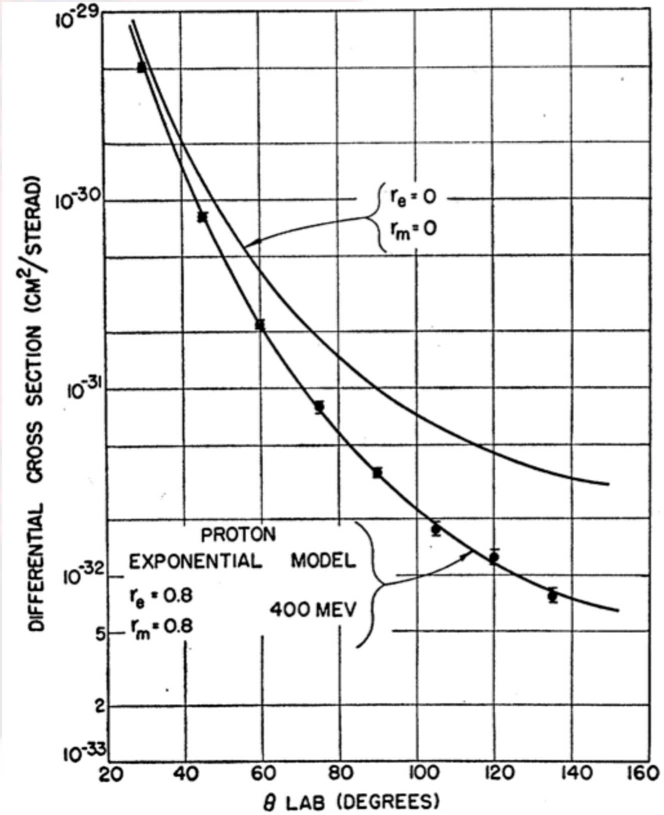
Rutherford, 1911

Finite size of nucleon (charge radius)

Hofstadter

- Rutherford scattering with electron





Renewed interest on proton radius:
μ-Atom vs e-Atom (EM-form factor)

RevModPhys.28.214

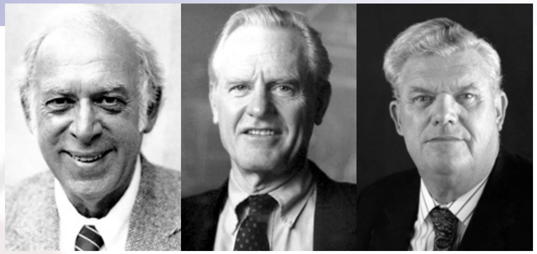
Quark model

Gell-Man

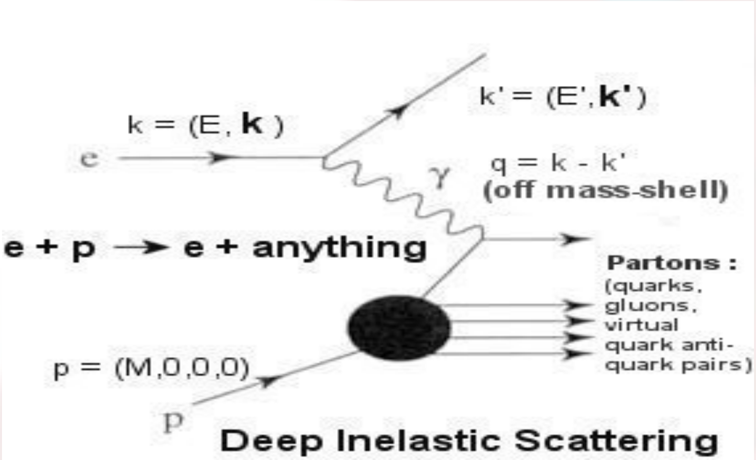
- Nucleons, and other hadrons are not fundamental particles, they have constituents
- Gell-Man Quark Model
  - Quark: spin ½
    - Charges: up (2/3), down (-1/3), strange (-1/3)
  - Flavor symmetry to classify the hadrons
    - Mesons: quark-antiquark
    - Baryons: three-quark
    - Gell-Man-Okubo Formula

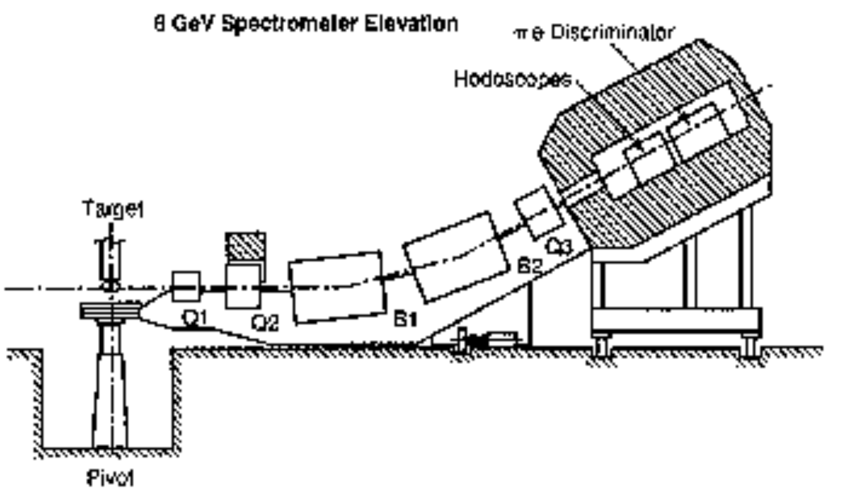
Deep Inelastic Scattering

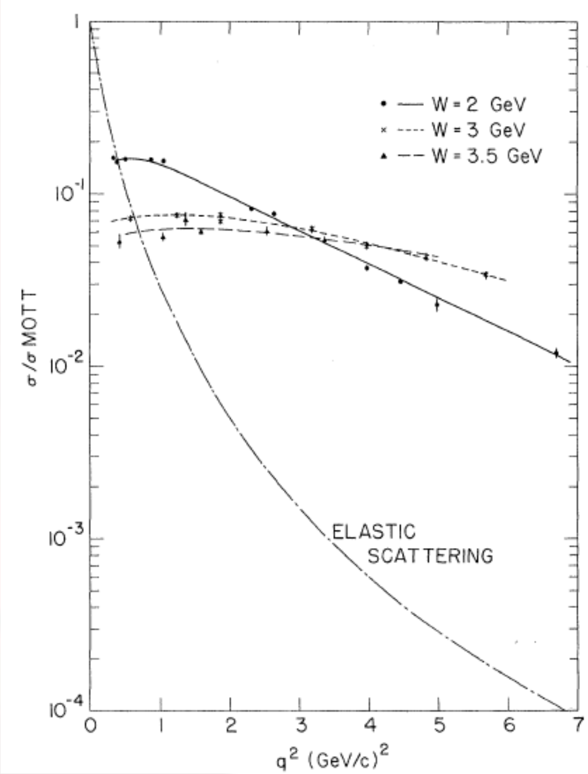
Discovery of Quarks



Friedman Kendall Taylor





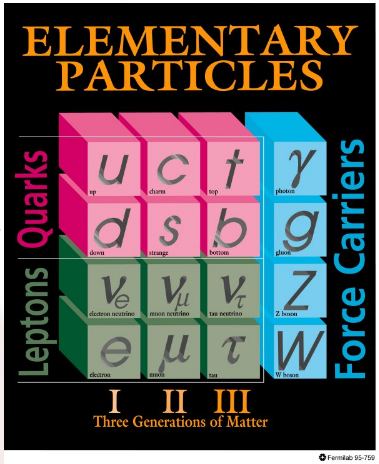


Bjorken Scaling: $Q^2 \rightarrow$ Infinity
Feynman Parton Model:
Point-like structure in Nucleon

Understanding the scaling

- Weak interactions at high momentum transfer
  - Rutherford formula rules
- Strong interaction at long distance
  - Form factors behavior
  - No free constituent found in experiment
- Strong interaction dynamics is different from previous theory

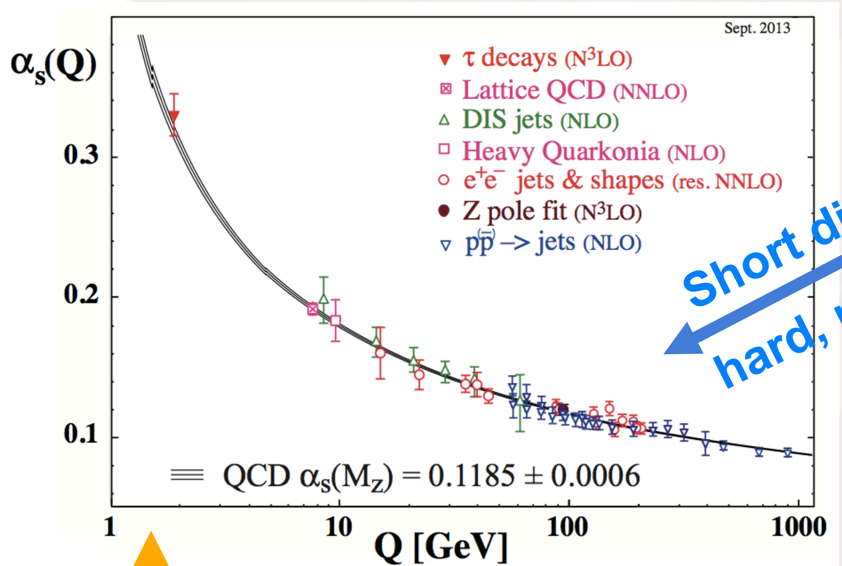
QCD and Strong-Interactions



- QCD: Non-Abelian gauge theory
  - Building blocks: quarks (spin$\frac{1}{2}$, m_q, 3 colors; gluons: spin 1, massless, 3^2-1 colors)

$$L = \bar{\psi}(i\gamma\cdot\partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma\cdot A\psi$$

- Asymptotic freedom and confinement

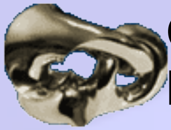


~1/Length

Nonperturbative scale Λ_{QCD}~1GeV

Short distance
hard, perturbative

Long distance:? Soft, non-perturbative



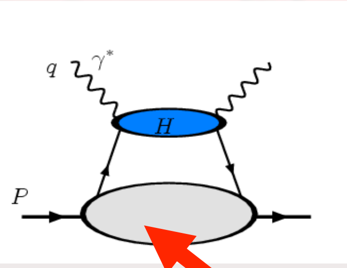
Clay Mathematics Institute
Millennium Prize Problem

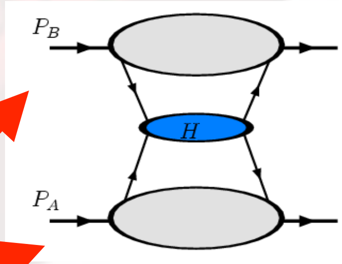
Quantum Chromodynamics

- There is no doubt that QCD is the right theory for hadron physics
- However, many fundamental questions…
- How does the **nucleon mass**?
- Why quarks and gluons are **confined** inside the nucleon?
- How do the fundamental **nuclear forces** arise from QCD?
- We don't have a **comprehensive picture** of the nucleon structure as we don't have an approximate QCD nucleon wave function
- …

Feynman's parton language and QCD Factorization

- If a hadron is involved in high-energy scattering, the physics simplifies in the infinite momentum frame (Feynman's Parton Picture)
- The scattering can be decomposed into a convolution of parton scattering and parton density (distribution), or wave function or correlations
  - QCD Factorization!

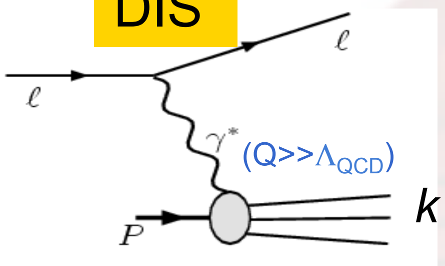




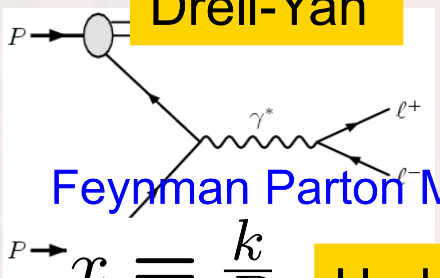
$$\sim \int \text{Parton Distributions} \otimes \text{Hard Partonic Cross Section}$$

High energy scattering as a probe to the nucleon structure

DIS



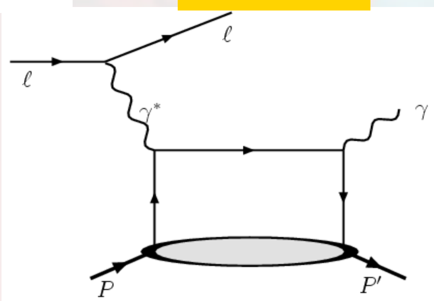
Drell-Yan



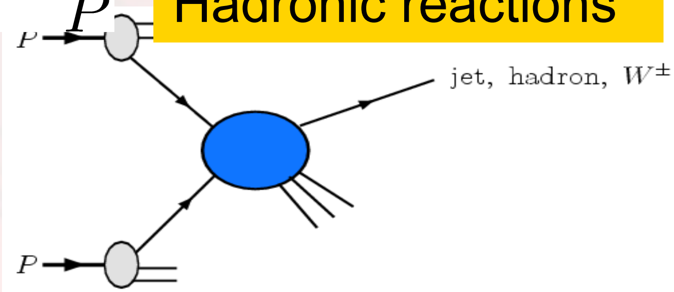
Feynman Parton Momentum fraction

$$x = \frac{k}{P}$$

DVCS

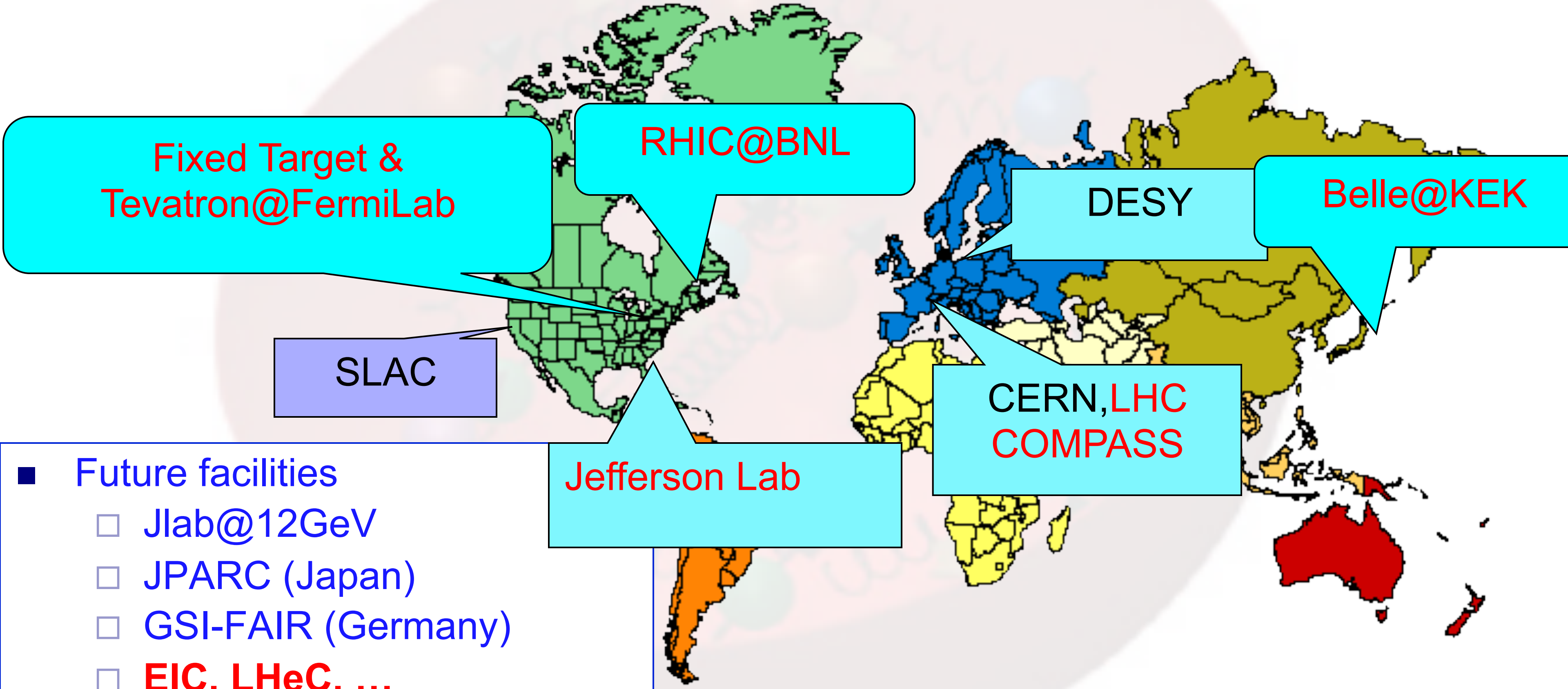


Hadronic reactions



- Many processes: Deep Inelastic Scattering, Deeply-virtual compton scattering, Drell-Yan lepton pair production, pp→jet+X
  - Momentum distribution: Parton Distribution
  - Spin density: polarized parton distribution
  - Wave function in infinite momentum frame: Generalized Parton Distributions

Exploring the partonic structure of nucleon worldwide



- Future facilities
  - Jlab@12GeV
  - JPARC (Japan)
  - GSI-FAIR (Germany)
  - **EIC, LHeC, …**

Perturbative corrections

- Singularities in higher order calculations
- Dimension regularization
  - n<4 for UV divergence
  - n>4 for IR divergence
  - MS ($\overline{\text{MS}}$) scheme for UV divergence

$$\int \frac{d^n k}{k^4} \to \int \frac{dk}{k} k^{n-4}$$

- pQCD predictions rely on Infrared safety of the particular calculation

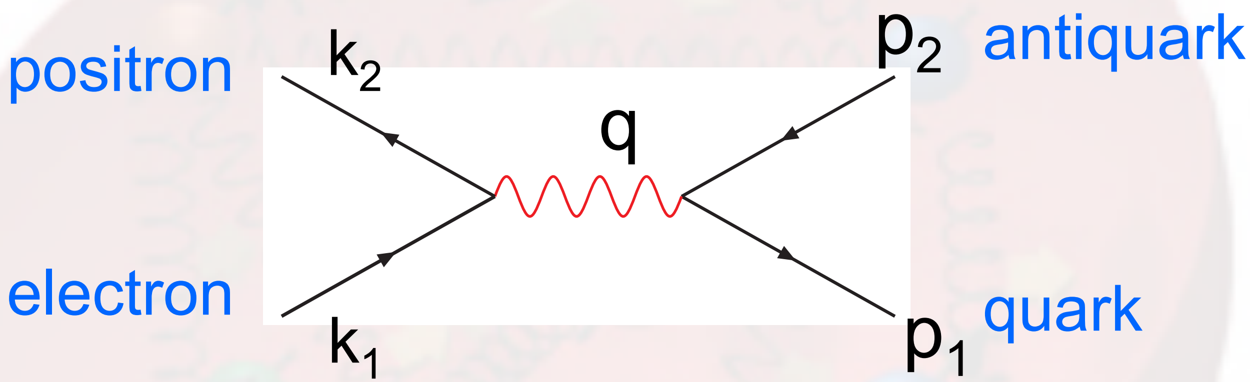
pQCD predictions

- Infrared safe observables
  - Total cross section in e+e-→hadrons
  - EW decays, tau, Z, …
- Factorizable hard processes: parton distributions/fragmentation functions
  - Deep Inelastic Scattering
  - Drell-Yan Lepton pair production
  - Inclusive process in ep, ee, pp scattering, W, Higgs, jets, hadrons, …

- Light-cone wave functions, factorization for the hard exclusive processes
  - Generalized Parton Distributions and form factors
- Effective theory
  - Heavy quark effective theory, heavy meson decays
  - Non-relativistic QCD, heavy quarkonium decay and production
- Soft-collinear effective theory

Infrared safe: $e^+e^- \rightarrow$ hadrons

- Leading order

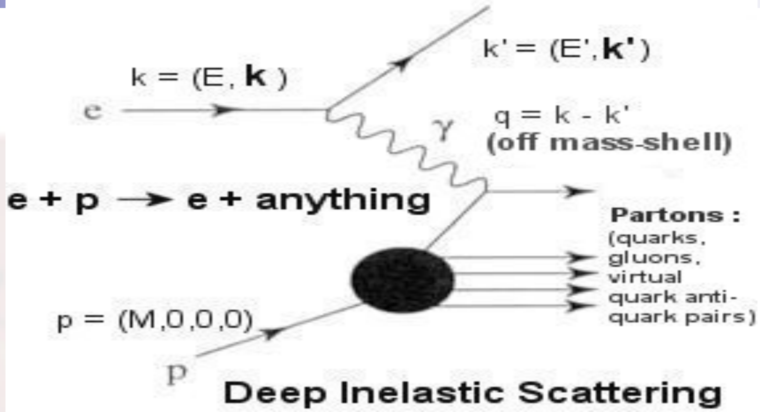


  - Electron-positron annihilate into virtual photon, and decays into quark-antiquark pair, or muon pair
  - Quark-antiquark pair hadronize

Long distance physics (factorization)

- Not every quantities calculated in perturbative QCD are infrared safe
  - Hadrons in the initial/final states, e.g.
- Factorization guarantee that we can safely separate the long distance physics from short one
- There are counter examples where the factorization does not work

Back to DIS



- Kinematics

$\nu = \dfrac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the nucleon rest frame (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the nucleon rest frame.

$Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}\,') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell(m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle with respect to the lepton beam direction.

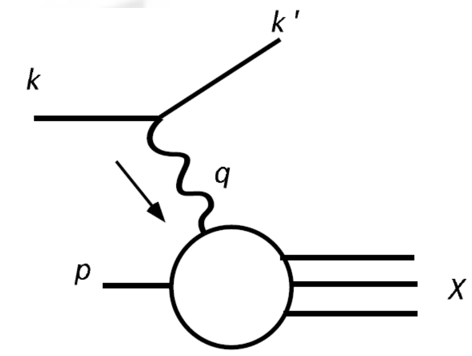
$x = \dfrac{Q^2}{2M\nu}$ where, in the parton model, x is the fraction of the nucleon's momentum carried by the struck quark.

$y = \dfrac{q \cdot P}{k \cdot P} = \dfrac{\nu}{E}$ is the fraction of the lepton's energy lost in the nucleon rest frame.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system X recoiling against the scattered lepton.

$s = (k + P)^2 = \dfrac{Q^2}{xy} + M^2 + m_\ell^2$ is the center-of-mass energy squared of the lepton-nucleon system.

Structure functions (cross section)



- EM factorization (photon exchange)

$$d\sigma = \frac{d^3k'}{2s|\vec{k}'|}\,\frac{1}{(q^2)^2}\,L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

$$L^{\mu\nu} \equiv \frac{e^2}{8\pi^2} tr\left[\not{k}\gamma^\mu \not{k}'\gamma^\nu\right]$$

- Hadronic tensor

$$W_{\mu\nu} \equiv \frac{1}{8\pi}\sum_{\text{spins }\sigma}\sum_{X} < N(p,\sigma) \mid J_\mu(0) \mid X > < X \mid J_\nu(0) \mid N(p,\sigma) > \times (2\pi)^4\delta^4(p_X - q - p)\,.$$

- Symmetry property for hadronic tensor
  - Spin average $W_{\mu\nu}^{(\text{em})} = W_{\nu\mu}^{(\text{em})}$
  - Time-reversal invariance $W_{\mu\nu} = W_{\mu\nu}^*$
  - Current conservation $q^\mu W_{\mu\nu} = 0$
  - Two independent structure functions

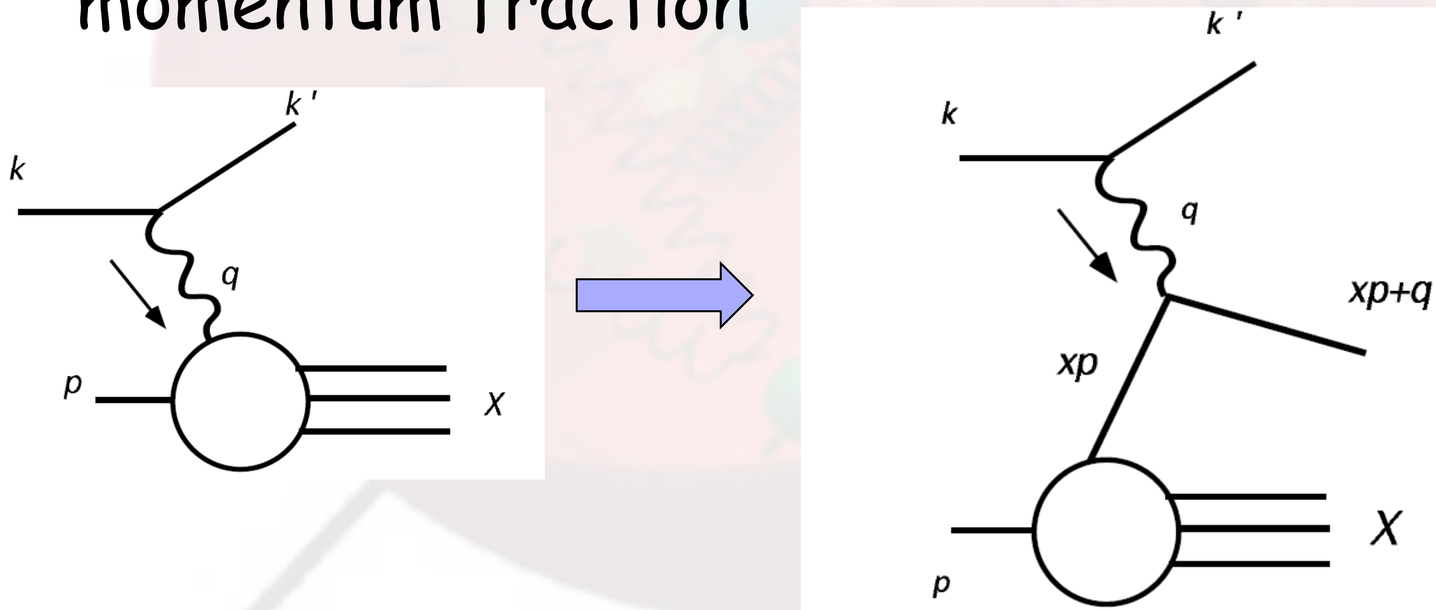
$$
\begin{aligned}
W_{\mu\nu}{}^{(em)} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x, q^2) \\
&\quad + \left(p_\mu + q_\mu\left(\frac{1}{2x}\right)\right)\left(p_\nu + q_\nu\left(\frac{1}{2x}\right)\right) W_2(x, q^2)
\end{aligned}
$$

$$
= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x, Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P \cdot q} F_2(x, Q^2) \qquad \hat{P}_\mu = P_\mu - \frac{P \cdot q}{q^2} q_\mu
$$

Naïve Parton Model

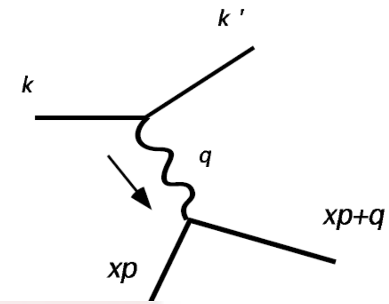
$$d\sigma^{(\ell N)}(p,q) = \sum_f \int_0^1 d\xi \; d\sigma_{\text{Born}}{}^{(\ell f)}(\xi p, q)\phi_{f/N}(\xi)$$

- $\phi_{f/N}(\xi)$ the parton distribution describes the probability that the quark carries nucleon momentum fraction



Naïve Parton Model

$$d\sigma^{(\ell N)}(p,q) = \sum_f \int_0^1 d\xi \; d\sigma_{\mathrm{Born}}{}^{(\ell f)}(\xi p, q)\phi_{f/N}(\xi)$$



- Partonic tensor is calculated

$$W_{\mu\nu}{}^{(f)} = \frac{1}{8\pi}\int \frac{d^3p'}{(2\pi)^3 2\omega_{p'}} Q_f{}^2 tr[\gamma_\mu \not{p}' \gamma_\nu \not{p}]\,(2\pi)^4\delta^4(p'-\xi p - q)$$

- Structure functions

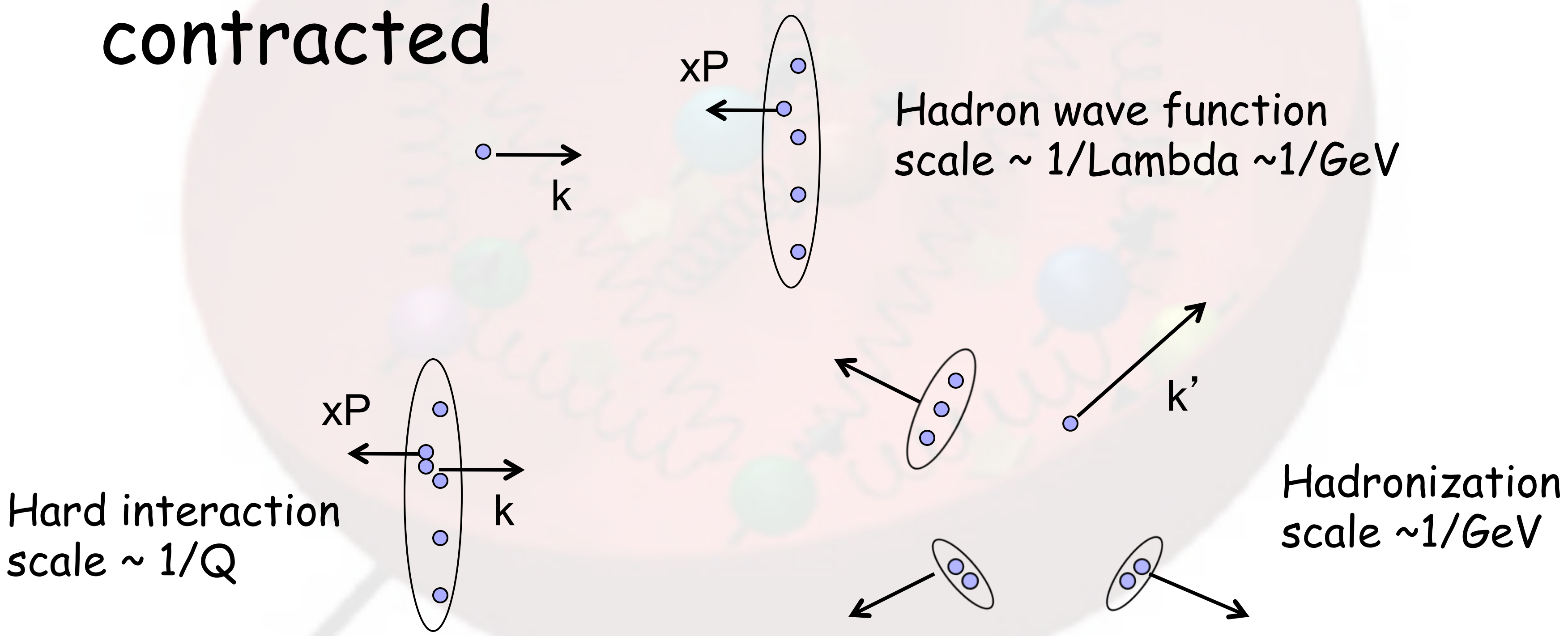
$$F_2{}^{(N)}(x) = \sum_f Q_f{}^2 x\phi_{f/N}(x) = 2xF_1{}^{(N)}(x)$$

  - Callan-Gross relation:
  - Quark spin is $\frac{1}{2}$

$$F_2 = 2xF_1$$

Intuitive argument for the factorization (DIS)

- In the Bjorken limit, nucleon is Lorentz contracted



Factorization formula

$$F_2{}^{(h)}(x,Q^2) = \sum_{i=f,\bar f,G} \int_x^1 d\xi \; C_2{}^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

$$F_1{}^{(h)}(x,Q^2) = \sum_{i=f,\bar f,G} \int_x^1 \frac{d\xi}{\xi} \; C_1{}^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

- Factorization → scale dependence

$$\mu\frac{d^2}{d\mu^2}\phi_{i/h}(x,\mu^2) = \sum_{j=f,\bar f,G}\int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi},\alpha_s(\mu^2)\right)\phi_{j/h}(\xi,\mu^2)$$

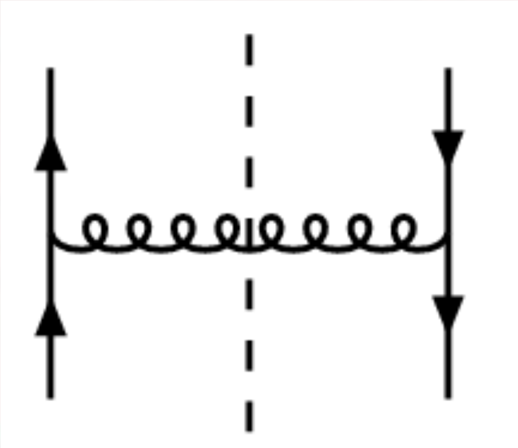
$$\mu\frac{d}{d\mu}\ln\bar\phi\left(n,\alpha_s(\mu^2)\right) = -\gamma_n\left(\alpha_s(\mu^2)\right) \qquad \bar f(n) \equiv \int_0^1 dx\; x^{n-1} f(x)$$

- Scale dependence → resummation

$$\bar\phi^{(\mathrm{val})}(n,\mu^2) = \bar\phi^{(\mathrm{val})}(n,\mu_0{}^2)\,\exp\left\{-\frac{1}{2}\int_0^{\ln\mu^2/\mu_0{}^2} dt\;\gamma_n\left(\alpha_s(\mu_0{}^2 e^t)\right)\right\}$$

anomalous dimension: $\int_0^1 d\xi\,\xi^{n-1}P_{ij}(\xi,\alpha_s) = -\gamma_{ij}(n)$

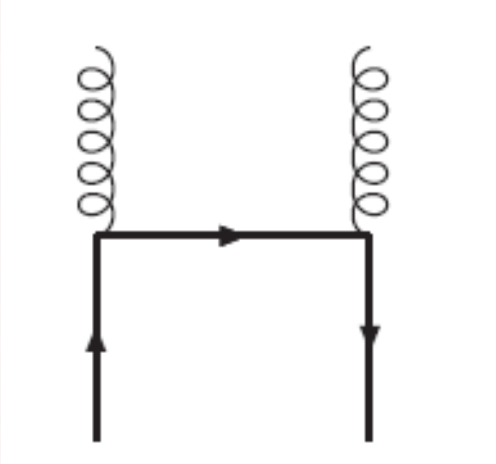
Quark-quark splitting



- Physical polarization for the radiation gluon
- Incoming quark on-shell, outgoing quark off-shell

$$\mathcal{P}_{qq} = C_F \left[\frac{1+x^2}{(1-x)_+} + \delta(1-x) \right]$$

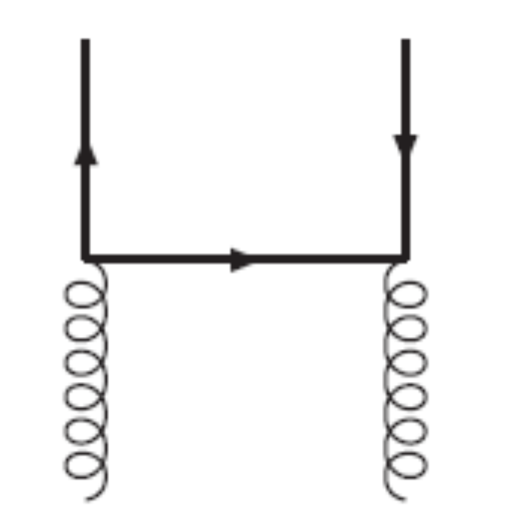
Quark-gluon splitting



- Incoming quark on-shell, gluon is off-shell

$$\mathcal{P}_{g/q} = C_F \left[\frac{1 + (1-x)^2}{x} \right]$$

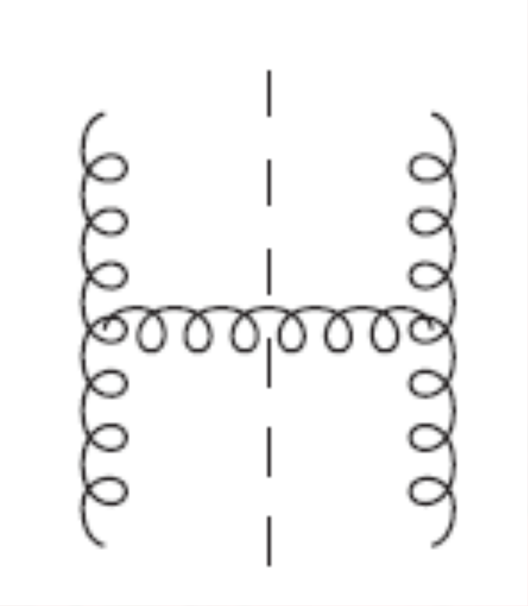
Gluon-quark splitting



- Incoming gluon is on-shell, physical polarization

$$\mathcal{P}_{q/g} = T_F \left[(1-x)^2 + x^2 \right]$$

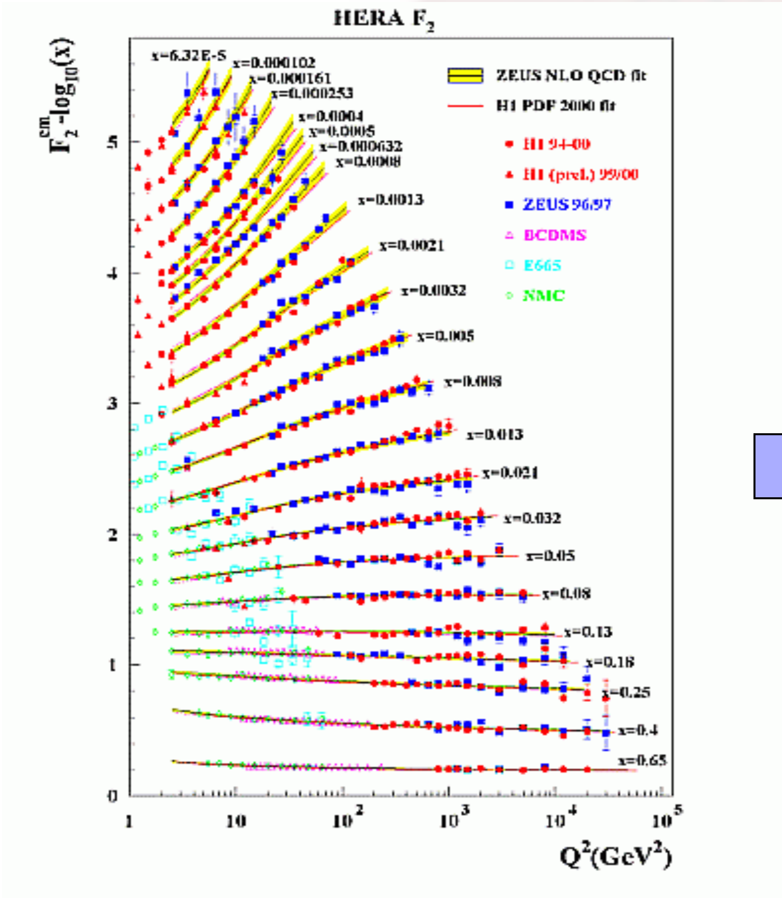
Gluon-gluon splitting

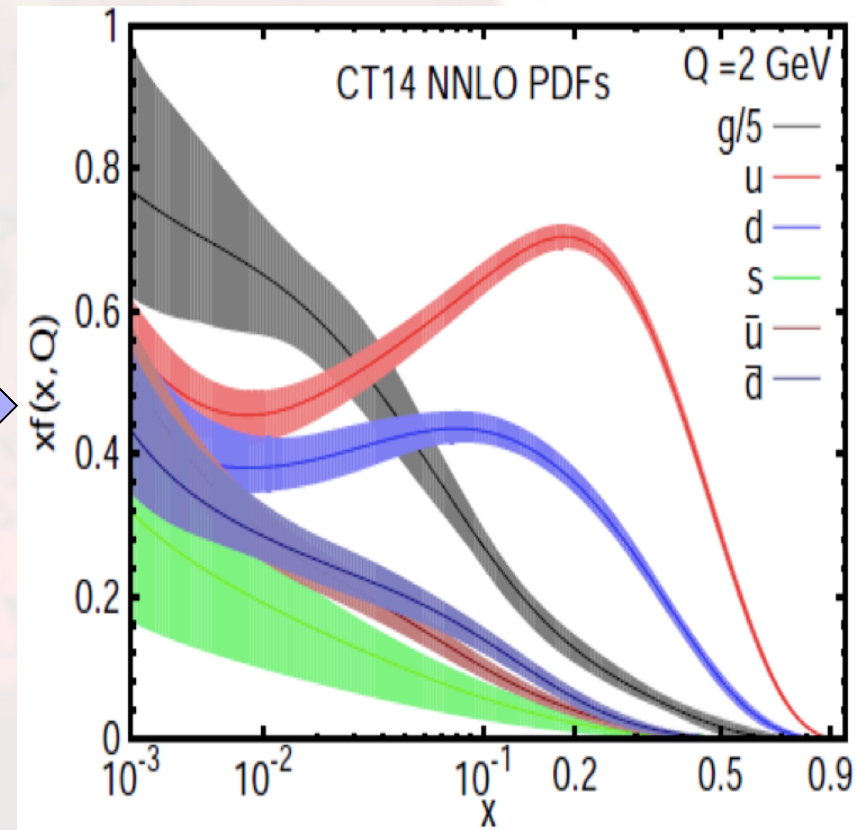


- Physical polarizations for the gluons

$$\mathcal{P}_{gg}(x) = \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \delta(x-1)\beta_0$$

These evolutions describe the HERA data





Reverse the DIS: Drell-Yan

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

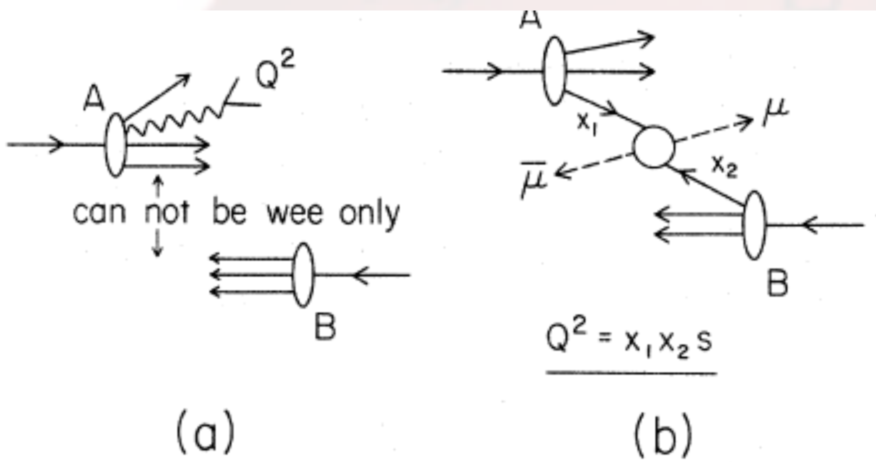
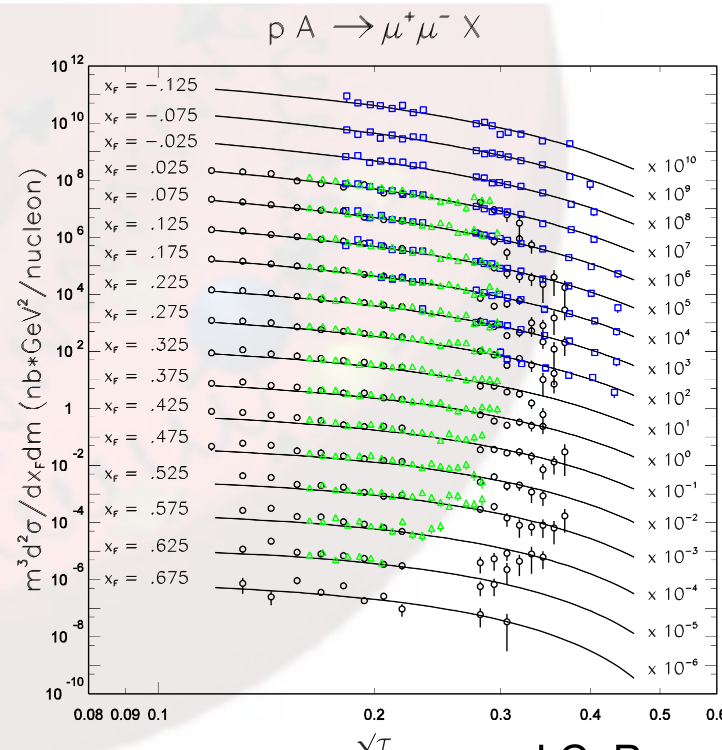
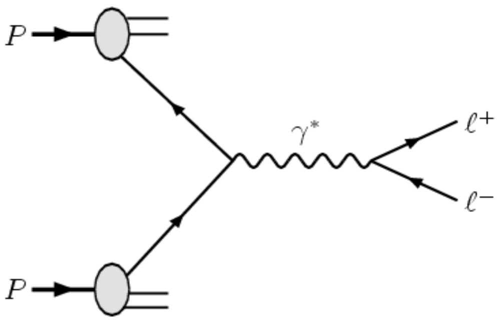


FIG. 1. (a) Production of a massive pair Q^2 from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.



J.C. Peng

Drell-Yan lepton pair production



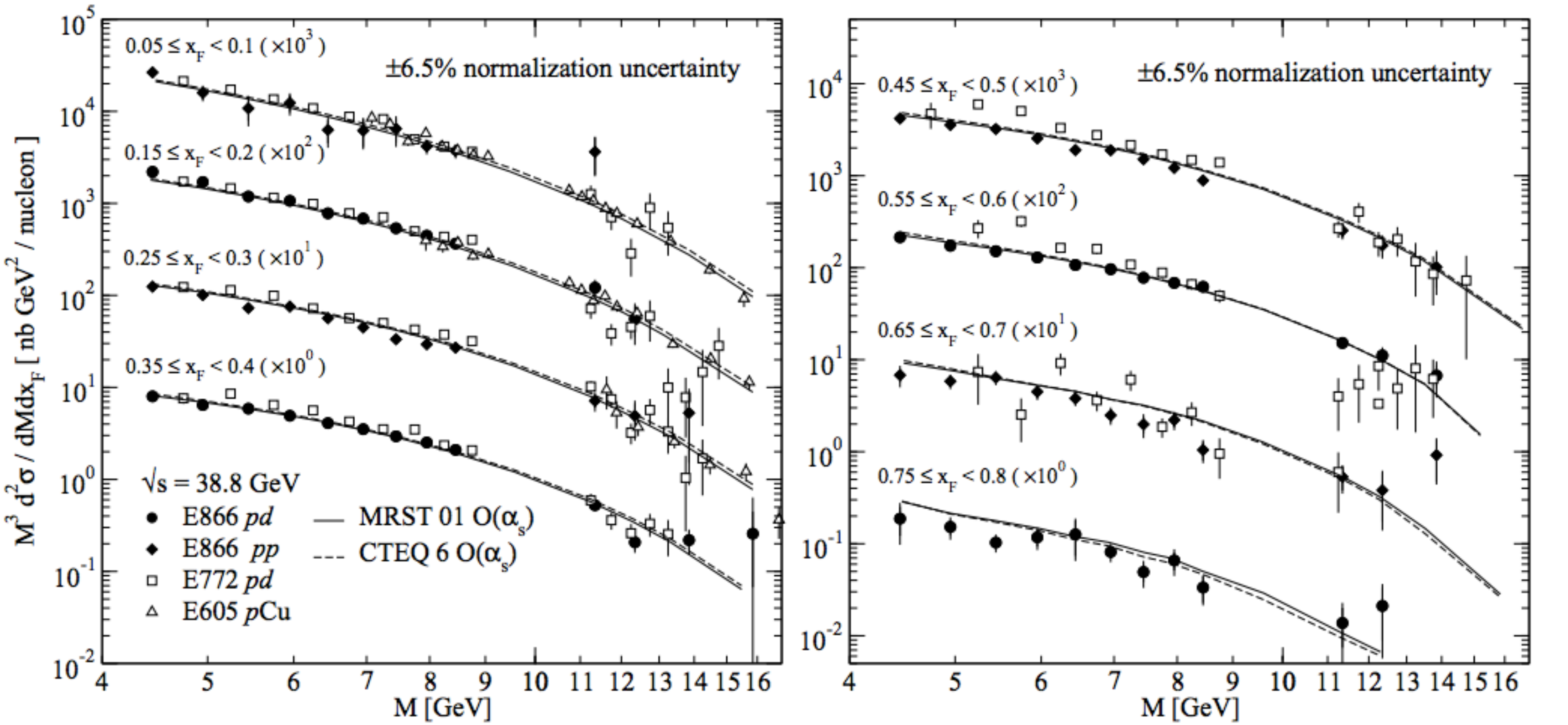
$$\sigma(pp \to \ell^+\ell^- + X) = \int dx_1 dx_2 \phi_{q/p}(x_1)\phi_{\bar{q}/p}(x_2)\hat{\sigma}(q\bar{q} \to \ell^+\ell^-)$$

- The same parton distributions as DIS
  - Universality
- Partonic cross section

$$\sigma(e^+e^- \to q\bar{q}) = N_c \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2$$

$$\hat{\sigma}(q\bar{q} \to \ell^+\ell^-) = \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2\frac{1}{N_c}$$

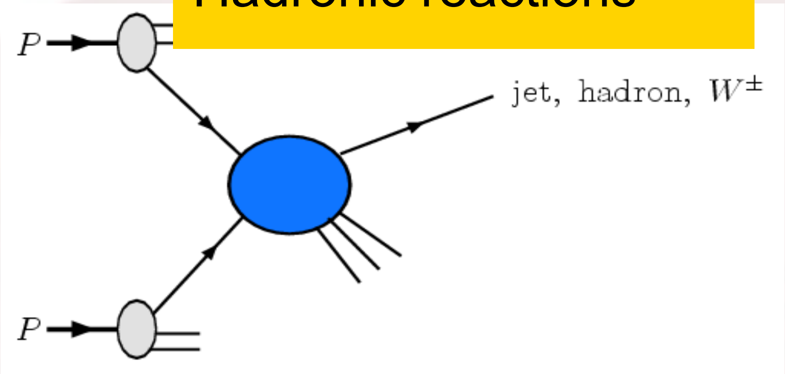
Profound results



- Universality
- Perturbative QCD at work

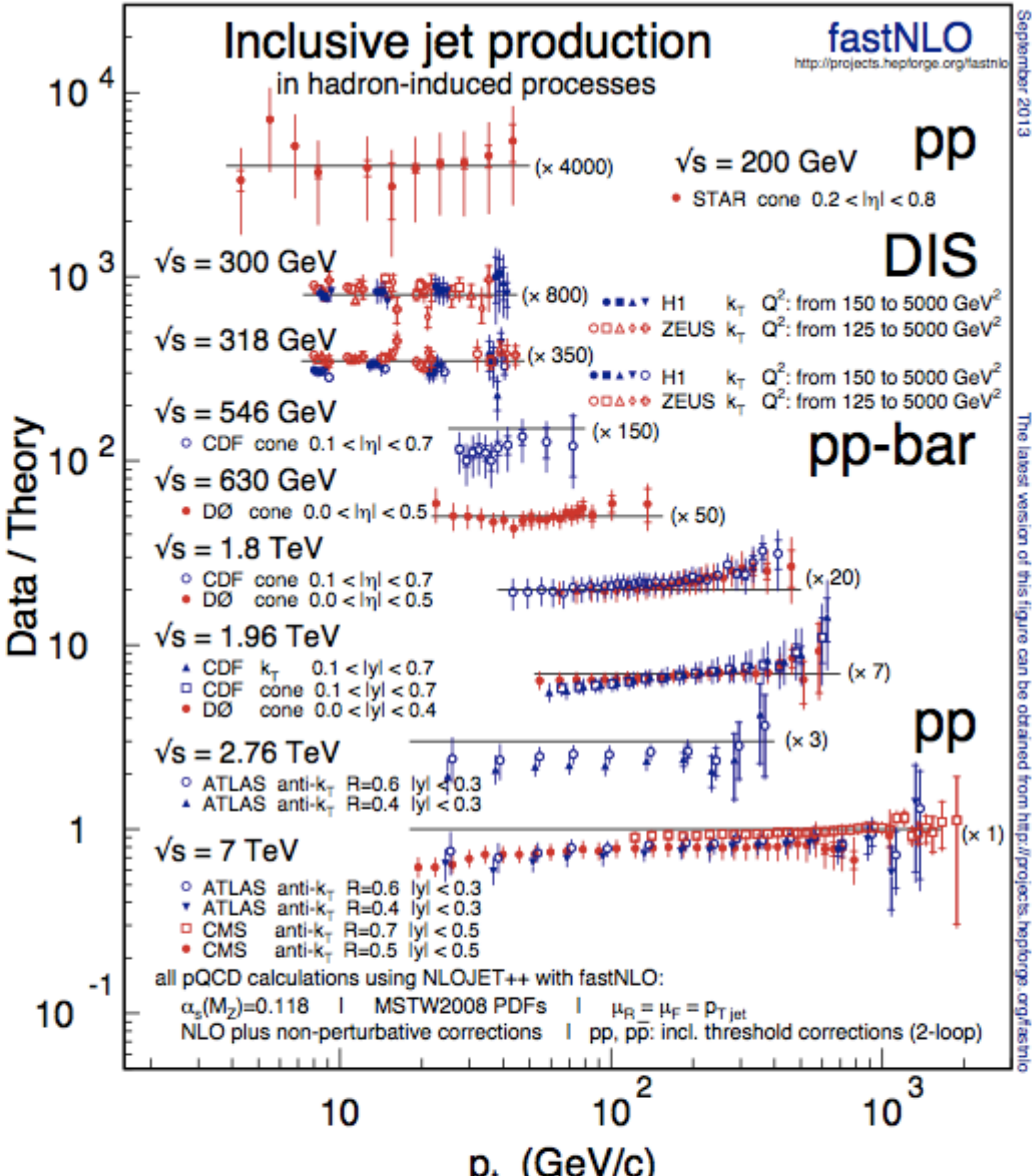
More general hadronic process

Hadronic reactions



$$\sigma(pp \to c + X) = \int dx_1 dx_2 \phi_{a/p}(x_1)\phi_{b/p}(x_2)\hat{\sigma}(ab \to c + X)$$

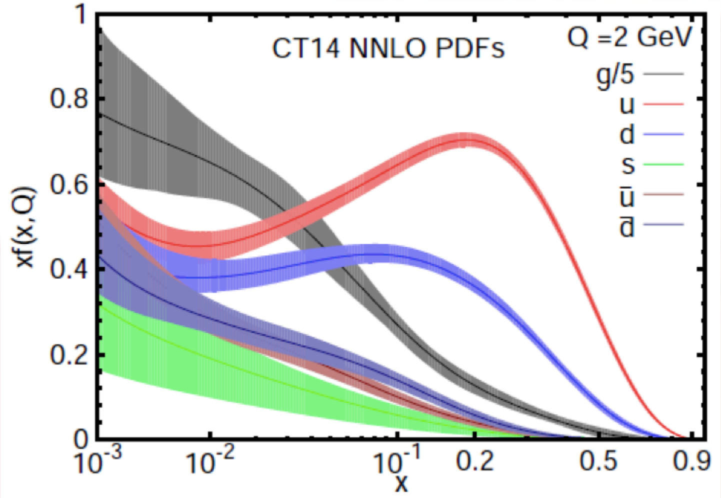
- All these processes have been computed up to next-to-leading order, some at NNLO, few at N^{3}LO



Inclusive jet production
in hadron-induced processes
fastNLO
http://projects.hepforge.org/fastnlo
September 2013
pp
√s = 200 GeV
STAR cone 0.2 < |η| < 0.8
(× 4000)
DIS
√s = 300 GeV
(× 800)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
√s = 318 GeV
(× 350)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
pp-bar
√s = 546 GeV
CDF cone 0.1 < |η| < 0.7
(× 150)
√s = 630 GeV
DØ cone 0.0 < |η| < 0.5
(× 50)
√s = 1.8 TeV
CDF cone 0.1 < |η| < 0.7
DØ cone 0.0 < |η| < 0.5
(× 20)
√s = 1.96 TeV
CDF k_T 0.1 < |y| < 0.7
CDF cone 0.1 < |y| < 0.7
DØ cone 0.0 < |y| < 0.4
(× 7)
pp
√s = 2.76 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
(× 3)
√s = 7 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
CMS anti-k_T R=0.7 |y| < 0.5
CMS anti-k_T R=0.5 |y| < 0.5
(× 1)
all pQCD calculations using NLOJET++ with fastNLO:
α_s(M_Z)=0.118 | MSTW2008 PDFs | μ_R = μ_F = p_T jet
NLO plus non-perturbative corrections | pp, pp̄: incl. threshold corrections (2-loop)
The latest version of this figure can be obtained from http://projects.hepforge.org/fastnlo
Data / Theory
10⁴
10³
10²
10
1
10⁻¹
10
10²
10³
p_t (GeV/c)

PDG2014

Parton picture of the nucleon



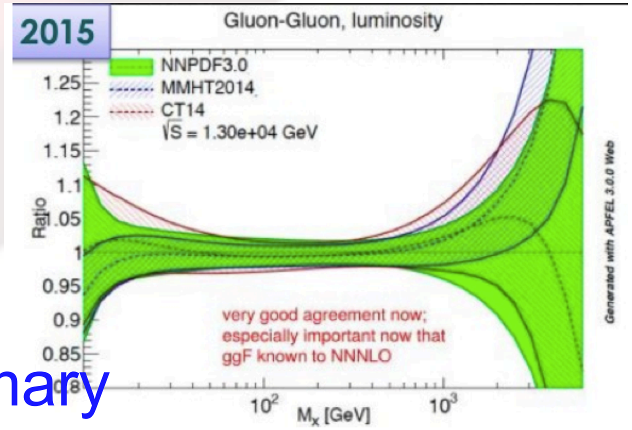
C.-P.Yuan@DIS15

- Beside valence quarks, there are sea and gluons
- Precisions on the PDFs are very much relevant for LHC physics: SM/New Physics

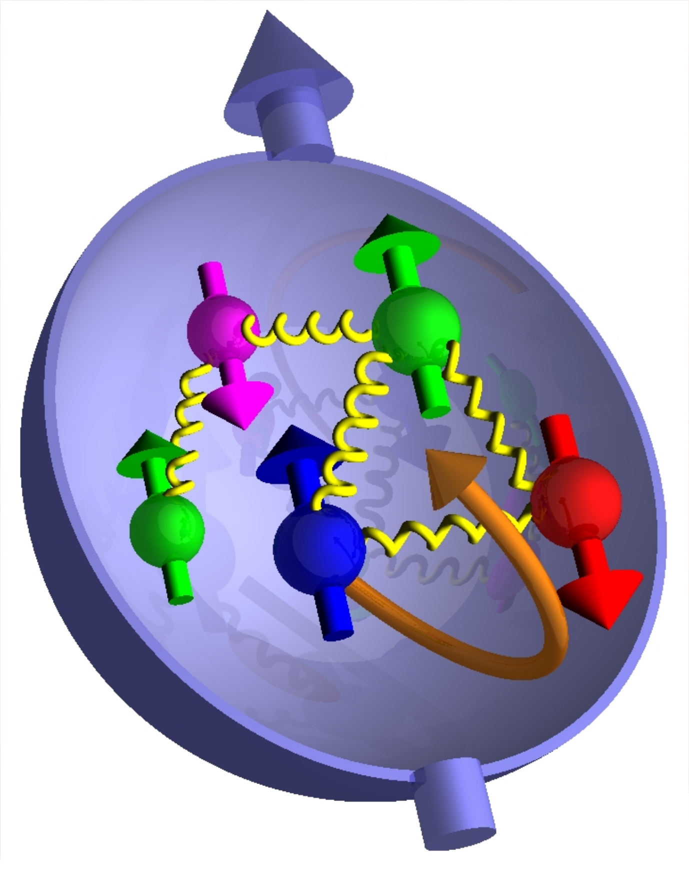
$$\sigma(gg \to H), \sqrt(s) = 13\text{TeV}$$

CT14	MMHT2014	NNPDF3.0
42.68 pb	42.70 pb	42.97 pb
+2.0% -2.4%	+1.3% -1.8%	+1.9% -1.9%

DIS summary



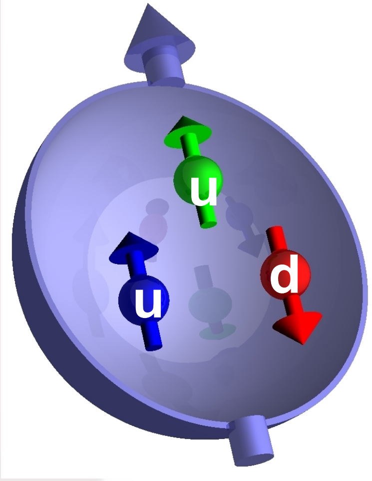
Parton distribution when nucleon is polarized?



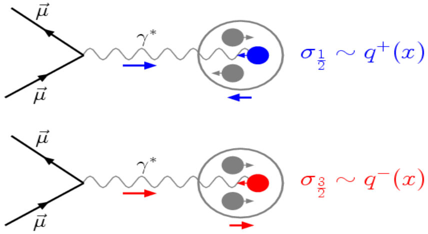
Proton Spin

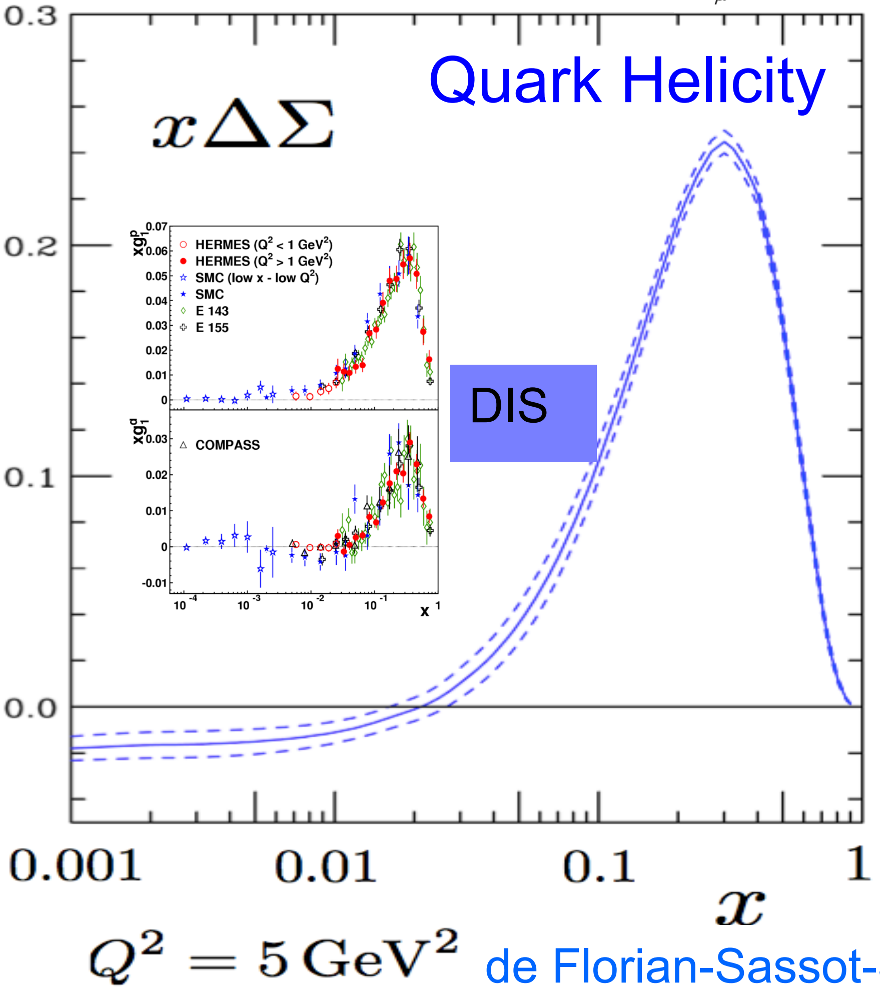
- The story of the proton spin began with the quark model in 60’s
- In the simple Quark Model, the nucleon is made of three quarks (nothing else)
- Because all the quarks are in the s-orbital, its spin ($\frac{1}{2}$) should be carried by the three quarks
- European Muon Collaboration: 1988

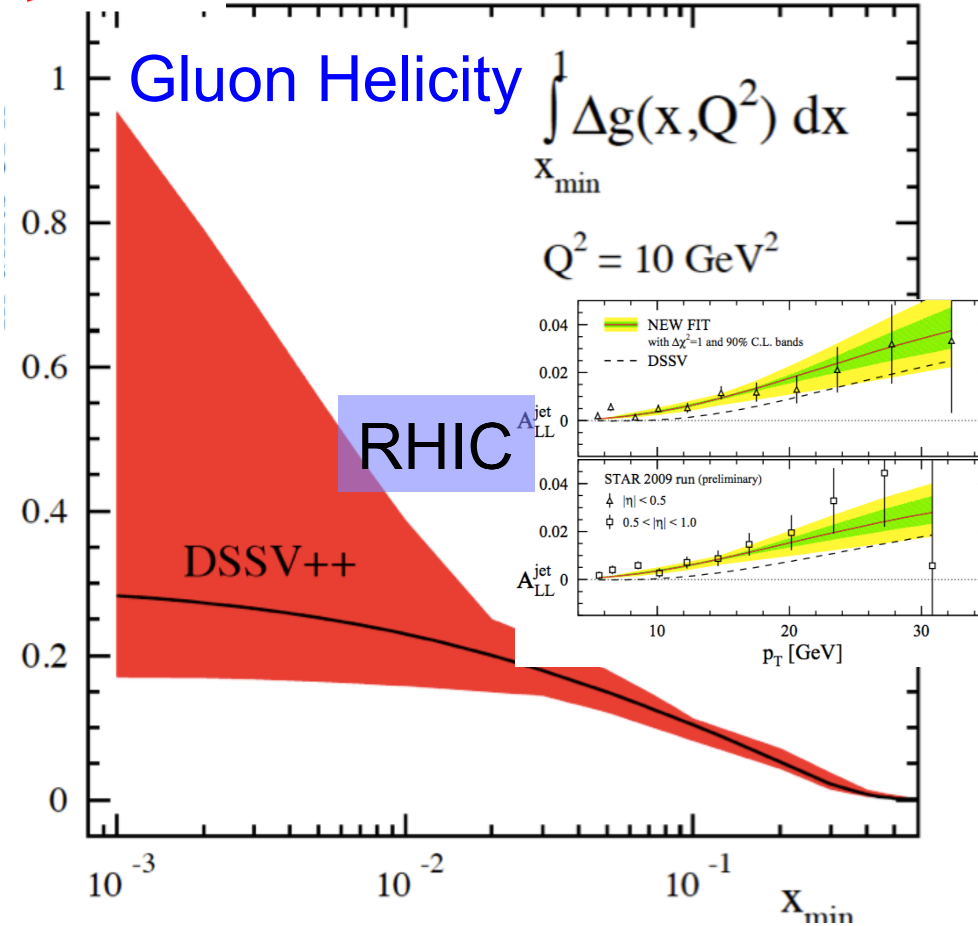
“Spin Crisis”--- proton spin carried by quark spin is rather small



Parton distributions in a polarized nucleon







de Florian-Sassot-Stratmann-Vogelsang, 2014

BERKELEY LAB

Proton spin: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$ emerging phenomena?

- We know fairly well how much quark helicity contributions, $\Delta\Sigma=0.3\pm0.05$
- With large errors we know gluon helicity contribution plays an important role
- No direct information on quark and gluon orbital angular momentum contributions

The orbital motion:

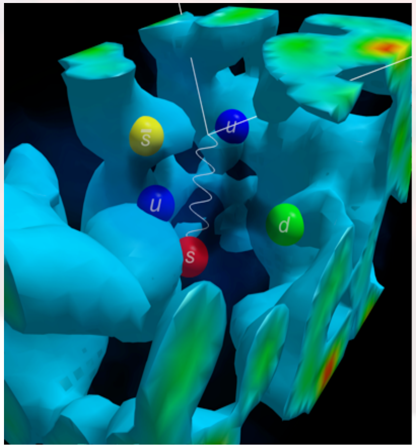
- Orbital motion of quarks and gluons must be significant inside the nucleons!
- Orbital motion shall generate direct orbital Angular Momentum which must contribute to the spin of the proton
- Orbital motion can also give rise to a range of interesting physical effects (Single Spin Asymmetries)

Theoretical Issues

- New structure, new dynamics and new phenomena!
  - New Structure and probe physics separation or factorization
  - New processes to measure novel observables
    - Spin correlation to study orbital motion
  - Study partons directly on lattice

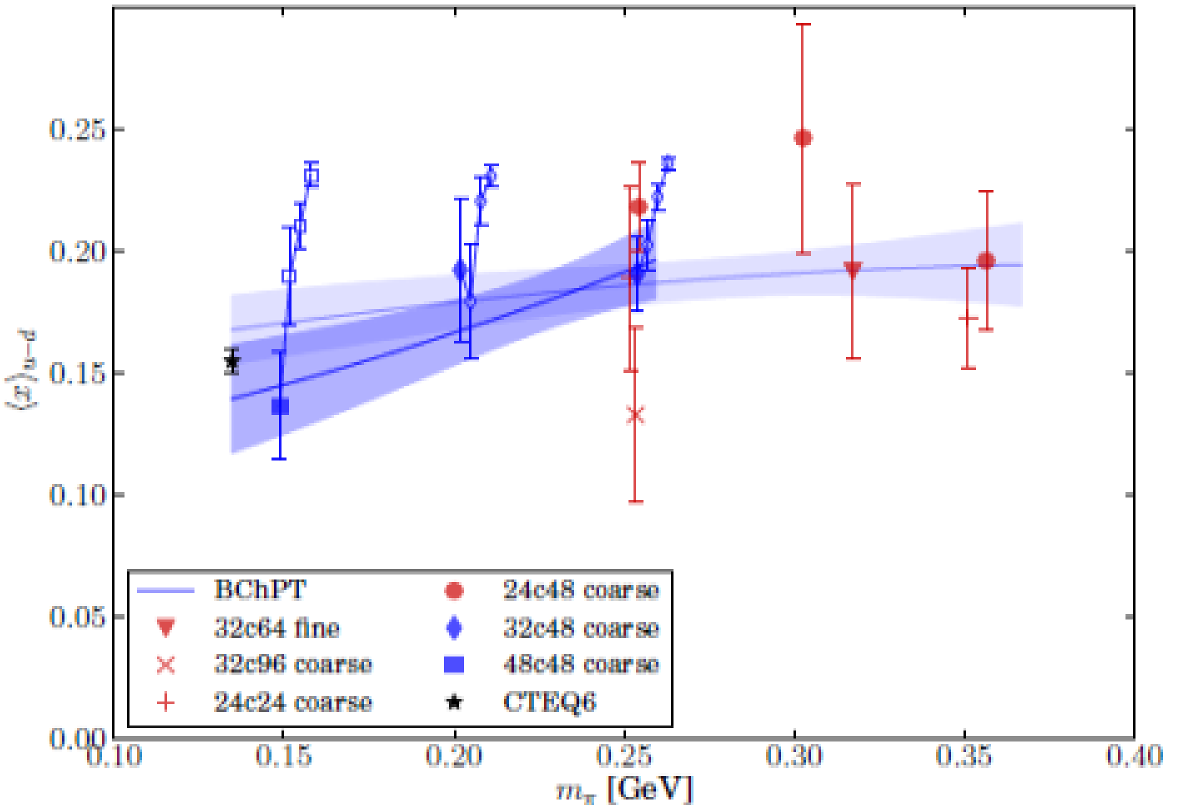
Lattice QCD

- The only known rigorous framework for *ab-initio* calculation of the structure of protons and neutrons with controllable errors.
- After decades of effort, one can finally calculate nucleon properties with dynamical fermions at physical pion mass!



Nucleon Structure from Lattice QCD

J.R. Green et al, 2012 & 2014

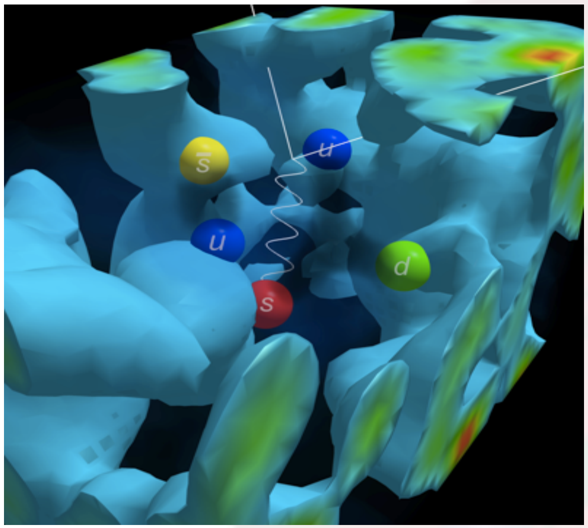


Nearly physical pion mass m_π=149MeV

Quark momentum fraction

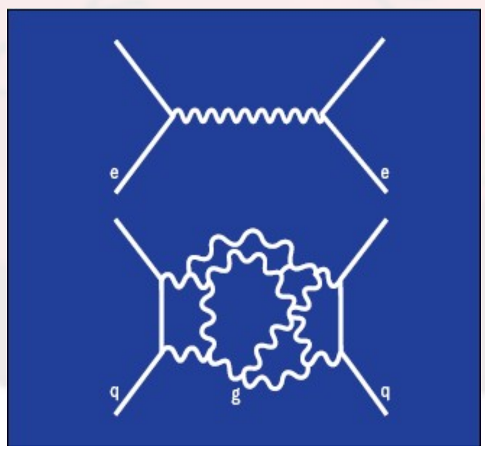
$$\langle x \rangle_{u-d} = \int dx\, x\, (u + \bar{u} - d - \bar{d})$$

Fundamental Understanding of the Nucleon Structure in QCD



Lattice QCD

EXP. Measurements



Theory/ Phenomenology

Partonic cross section eq→e'q'

- Cross symmetry with e+e-→qq

$$d\sigma = \frac{d^3k'}{2s|\vec{k}'|}\frac{1}{(q^2)^2}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

$$L^{\mu\nu} \equiv \frac{e^2}{8\pi^2}tr\left[\not{k}\gamma^\mu\not{k}'\gamma^\nu\right]$$

$$|\overline{\mathcal{M}}|^2 = \frac{1}{(q^2)^2}L_{\mu\nu}W_{\mu\nu} = e_q^2\frac{e^4}{(q^2)^2}2\left[s^2+u^2\right]$$

$$u = (k'-p)^2 = -2k'\cdot p = -s(1-y), \quad y = \frac{q\cdot p}{k\cdot p}$$

$$(s^2+u^2) = s^2(1+(1-y)^2)$$

$$d\sigma(ep\to e'+X) = \int dxdy\frac{2\pi\alpha^2}{Q^2}\left[1+(1-y)^2\right]\sum_q e_q^2\phi_{q/P}(x)$$